pjk, j ∈ ℕ , k ∈ M, be the baseline processing time of job j on machine k

djk,­ j ∈ ℕ , k ∈ M, the deteriorating delay in completion time of job j on machine k with djk ≥ 1

The problem under consideration is the assignment of jobs to the machines, the sequencing of the jobs on the machines, and the schedule of the maintenance activities so that the makespan i.e, the maximum completion time CMAX of all the jobs is minimized. Let C­MAX = ­maxk ∈ M {Ck} where Ck is the sum of the actual processing times for the jobs assigned to the machine and the duration of the maintenance activities. This problem is NP-hard. In the absence of machine maintenance and deterioration it reduces to R||CMAX problem, which is NP-Hard (Pinedo 2012)

The mathematical formulation for this problem is presented next. Let H be the possible number of position in each machine where H = {1,2,…, 2(n-m) +1}. The first decision variable xjkh, j in N, k ∈ M, h ∈ H, is a binary variable that is equal to 1 if job j is assigned to machine k in position h, 0 otherwise. The second decision variable skh, k ∈ M, h ∈ H, is a binary variable that is equal to 1, if there is a maintenance event in machine k in position h, 0 otherwise. The third decision variable is qkh , k ∈ M, h ∈ H, which is the delay on the completion time of machine k for the job in position h. Using these three decision variables, the problem can be modelled as follows.

*New variables*

αjkh 🡪 Linearized total completion time of job j ∈ N on machine k ∈ M executed on position h ∈ H